NativityMiguel Network of Schools

Standards and Benchmarks in Mathematics for an 8th Grade Graduate

(Adapted 2006)

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Students in member schools of the NativityMiguel Network will expand and deepen their knowledge of numbers, computation, estimation, measurement, patterns and functions, and the fundamental concepts of data analysis, probability, geometry and algebra by focusing on meaningful mathematics in each of these areas. It is critical that students demonstrate mastery in number sense and operations and basic understanding of measurement and problem solving before moving into more complex concepts, such as data analysis, probability, geometry and algebra, which will be constructed upon prior competencies.

Instruction will include activities in which the students actively work to pose and solve problems both individually and collectively. Beneficial learning tools, such as concrete models, fraction manipulatives, algebra tiles, and geoboards, will be available to students. The appropriate use of calculators and computers will also be taught. Graduates are also expected to exhibit facility with mental arithmetic.

Graduates are expected to demonstrate competence in the general skills and strategies of the following process standards:

I. Number Sense and Operations

Instruction in rational numbers will build on students' prior knowledge of whole-number concepts and skills and their encounters with fractions, decimals, and percents in lower grades and in everyday life. Students will become proficient in using fractions, decimals, percents, and integers to solve problems. By solving problems that require multiplicative comparisons (e.g., "How many times as many?" or "How many per?"), students will gain extensive experience with ratios, rates, and percents, which helps form a solid foundation for their understanding of, and facility with, proportionality.

To demonstrate competency in Number Sense and Operations students will:

A. Show evidence of appropriate use of basic operations in solving problems;
B. Apply the correct order of operations for arithmetic computations;
C. Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers;
D. Recognize and generate equivalent forms of commonly used fractions, decimals, and percents;
E. Work flexibly with fractions, decimals, and percents to solve problems;
F. Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line;
G. Develop meaning for percents greater than 100 and less than 1;
H. Understand and use ratios and proportions to represent quantitative relationships;
I. Develop an understanding of large numbers, recognize and appropriately use exponential, scientific, and calculator notation;
J. Use factors, multiples, prime factorization, and relatively prime numbers to solve problems;
K. Develop meaning for integers and represent and compare quantities with them;

L. Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals;
M. Demonstrate an understanding of root extraction;
N. Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems;
O. Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators, and paper and pencil;
P. Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use;
Q. Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results;
R. Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

II. Problem Solving

Instruction in Problem Solving will promote mathematical learning. Students will learn about and deepen their understanding of mathematical concepts by working through carefully selected problems that allow applications of mathematics to other contexts. Interesting problems will be suggested by everyday experiences, such as reading literature or using cellular telephones, in-line skates, kites, and paper airplanes for example. Instruction will take advantage of the expanding mathematical capabilities of students to include more-complex problems that integrate such topics as probability, statistics, geometry, and rational numbers. Situations and approaches will build on and extend the mathematical understanding, skills, and language that students have acquired.

To demonstrate competency in Problem Solving students will:

A. Develop and test strategies to solve practical, everyday problems which may have single or multiple answers;
B. Use technology to generate and analyze data to solve problems;
C. Assess a problem, determine information required to solve the problem, choose methods for obtaining information, and evaluate results to determine their reasonableness;
D. Apply a variety of strategies (e.g., restate the problem, look for a pattern, diagrams, solve a simpler problem, work backwards, trial and error) to solve problems, with emphasis on multi-step and non-routine problems;
E. Use oral, written, concrete, pictorial, graphical and/or algebraic methods to model mathematical situations;
F. Know how to estimate and when an estimate is more appropriate than an exact answer

III. Measurement

Instruction of measurement will include choosing and using compatible units for the attributes being measured, estimating measurements, selecting appropriate units and scales on the basis of the precision desired, and solving problems involving the perimeter and area of two-dimensional shapes and the surface area and volume of three-dimensional objects. Students will become proficient at measuring angles and using ratio and proportion to solve problems involving scaling, similarity, and derived measures.

To demonstrate competency in Measurement students will:

A. Understand both metric and customary systems of measurement;
B. Understand relationships among units and convert from one unit to another within the same system;
C. Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume;

D. Use common benchmarks to select appropriate methods for estimating measurements;
E. Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision;
F. Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes;
G. Develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders;
H. Solve problems involving scale factors, using ratio and proportion;
I. Solve simple problems involving rates and derived measurements for such attributes as velocity and density.

IV. Data Analysis and Probability

Instruction in Data Analysis and Probability will help students answer complex questions, such as those concerning relationships among populations or samples and those about relationships between two variables within one population or sample. Toward this end, new representations will be added to the students' repertoire. Box plots, for example, allow students to compare two or more samples, such as the heights of students in two different classes. Scatterplots will allow students to study related pairs of characteristics in one sample, such as height versus arm span among students in one class. In addition, students will use and further develop their emerging understanding of proportionality in various aspects of their study of data and statistics.

To demonstrate competency in Data Analysis and Probability students will:

A. Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population;
B. Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots;
C. Find, use, and interpret measures of center and spread, including mean and interquartile range;
D. Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots;
E. Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken;
F. Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit;
G. Use conjectures to formulate new questions and plan new studies to answer them;
H. Understand and use appropriate terminology to describe complementary and mutually exclusive events;
I. Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations;
J. Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

V. Geometry

In geometry, students investigate relationships by drawing, measuring, visualizing, comparing, transforming, and classifying geometric shapes. Geometry provides a rich context for the development of mathematical reasoning, including inductive and deductive reasoning, making and validating conjectures, and classifying and defining geometric shapes.

To demonstrate competency in geometry students will:

A. Precisely describe, classify, and understand relationships among types of two- and three-dimensional shapes using their defining properties;
B. Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar shapes;
C. Create and critique inductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship;
D. Specify locations and describe spatial relationships using coordinate geometry and other representational systems;
E. Use coordinate geometry to represent and examine the properties of geometric shapes;
F. Use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides;
G. Apply transformations and use symmetry to analyze mathematical situations;
H. Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling;
I. Examine the congruence, similarity, and line or rotational symmetry of shapes using transformations;
J. Use visualization, spatial reasoning, and geometric modeling to solve problems;
K. Draw geometric shapes with specified properties, such as side lengths or angle measures;
L. Use two-dimensional representations of three-dimensional shapes to visualize and solve problems such as those involving surface area and volume;
M. Use geometric models to represent and explain numerical and algebraic relationships;
N. Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

VI. Algebra

Instruction in Algebra focuses on the study of algebraic concepts including operations with real numbers and polynomials, relations and functions, creation and application of linear functions and relations, and an introduction to nonlinear functions. The development of both a symbolic and graphical understanding of the mathematics is emphasized. The integration of statistics and geometry into the course helps students to develop a better understanding of how different concepts relate to one another. Students are also regularly asked to apply their understanding of the mathematics to real-world situations. Working on projects and learning to use technology appropriately are integral parts of the course. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

To demonstrate competency in Algebra students will:

A. Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
B. Relate and compare different forms of representation for a relationship;
C. Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations;
D. Develop an initial conceptual understanding of different uses of variables;

E. Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;
F. Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
G. Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations;
H. Model and solve contextualized problems using various representations, such as graphs, tables, and equations;
I. Use graphs to analyze the nature of changes in quantities in linear relationships.

VII. Communicating Mathematical Ideas

Instruction in Communication will include learning to communicate mathematical arguments and rationales, not just procedural descriptions or summaries (Yackel and Cobb 1996). Each student will be expected not only to present and explain the strategy he or she used to solve a problem but also to analyze, compare, and contrast the meaningfulness, efficiency, and elegance of a variety of strategies. Teachers will build a sense of community in middle-grades classrooms so students feel free to express their ideas honestly and openly, without fear of ridicule.

To demonstrate competency in Communicating Mathematical Ideas students will:

A. Use appropriate language, symbolism, and terminology when describing objects, relationships, mathematical solutions and rationale;
B. Reflect and justify reasoning in mathematical problem solving (e.g., convince, demonstrate, formulate) both orally and in writing;
C. Organize and accurately labels work;
D. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
E. Analyze and evaluate the mathematical thinking and strategies of others.

VIII. Reasoning

Instruction in and opportunity for reasoning on a regular basis with the teacher and with one another, will allow students to explain the basis for their conjectures and the rationale for their mathematical assertions. Through these experiences, students will become more proficient in using inductive and deductive reasoning appropriately.

To demonstrate competency in Reasoning students will:

A. Identify and extend patterns and uses experiences and observations to make suppositions;
B. Use counterexamples to disprove suppositions (e.g., all square are rectangles, but are all rectangles squares?);
C. Develop and evaluate mathematical arguments (e.g., agrees or disagrees with the reasoning of other classmates and explains why).

IX. Representation

Instruction in Representation is central to the study of mathematics. Students will develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations. Representations—such as physical objects, drawings, charts, graphs, and symbols—will also help students communicate their thinking.

To demonstrate competency in Representation students will:

A. Organize, represent, analyze data in graphs, charts, tables, plots and spreadsheets;
B. Select and use appropriate units and tools, depending on the degree of accuracy required, to find measurements;

C. Select, apply, and translate among mathematical representations to solve problems;
D. Use representations to model and interpret physical, social, and mathematical phenomena.

X. Connections

Thinking mathematically involves looking for Connections, and instruction in making Connections builds mathematical understanding. Without connections, students must learn and remember too many isolated concepts and skills. With connections, they will build new understandings on previous knowledge. The important mathematical foci in the middle grades—rational numbers, proportionality, and linear relationships—are all intimately connected, so as students encounter diverse new mathematical content, they have many opportunities to use and make connections.

To demonstrate competency in Making Connections students will:

A. Apply mathematical strategies to solve problems that arise from other disciplines and the real world.
B. Connect one area or idea of mathematics to another.
C. Recognize the importance of mathematics as a tool in a variety of careers and areas of interest and/or hobbies.
D. Create and use representations to organize, record, and communicate mathematical ideas;
E. Use representations to model and interpret physical, social, and mathematical phenomena.